The Dividend Discount Model
Mid-Term Discounting

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In this white paper we will revise the Dividend Discount Model (DDM) for mid-term discounting. The standard DDM assumes that cash flow is received at the end of each annual period. Mid-term discounting assumes that cash flow is received at various points within each annual period. To that end we will work through the following hypothetical problem...

Our Hypothetical Problem

Assume that we are tasked with calculating the enterprise value of ABC Company given the following model parameters...

Table 1: Model Parameters

Description	Value
Free cash flow at time zero	\$1,000.00
Annualized growth rate of free cash flow	5.00%
Annualized cost of capital	15.00%

Question 1: What is the enterprise value of ABC Company given annual discounting?
Question 2: What is the enterprise value of ABC Company given monthly discounting?
Question 3: What is the enterprise value of ABC Company given continuous discounting?

Revising The DDM Model

We will define the variable C_n to be free cash flow in period n. Using Table 1 above, the equation for free cash flow in period zero is...

$$C_0 = 1,000.00 \tag{1}$$

We will define the variable m to be the number of intraperiods within each annual discounting period n and the variable Δ to be intraperiod length in years. This statement in equation form is...

$$m = \text{Number of intraperiods within each annual discounting period } \text{ ...such that... } \Delta = \frac{1}{m} \tag{2}$$

We will define the variable g to be the periodic growth rate of free cash flow. Using Equation (2) above, the equation for the periodic free cash flow growth rate is...

$$g = \left(1 + \text{annual rate}\right)^{1/m} - 1 \text{ ...when... } m < \infty \;\Big|\; g = \ln\left(1 + \text{annual rate}\right) \text{ ...when... } m = \infty \tag{3}$$

We will define the variable k to be the periodic cost of capital. Using Equation (2) above, the equation for the periodic cost of capital is...

$$k = \left(1 + \text{annual rate}\right)^{1/m} - 1 \text{ ...when... } m < \infty \;\Big|\; k = \ln\left(1 + \text{annual rate}\right) \text{ ...when... } m = \infty \tag{4}$$

We will define the variable V_0 to be the present value of free cash flow at time zero (i.e. enterprise value) and the variable θ to be the discount factor. Using Equations (1), (3) and (4) above, the equation for enterprise value is... [2]

$$V_0 = \Delta\, C_0 \sum_{n=1}^{\infty} \theta^n \text{ ...when... } m < \infty \;\Big|\; V_0 = C_0 \int_0^{\infty} \text{Exp}\left\{ \left(g - k \right) t \right\} \delta t \text{ ...when... } m = \infty \tag{5}$$

We will define the variable Γ to be the cash flow valuation multiple. Using Equation (5) above, the equation for the cash flow valuation multiple is... [1]

$$\Gamma = \Delta \sum_{n=1}^{\infty} \theta^n = \Delta\, \frac{\theta}{1-\theta} \text{ ...when... } m < \infty \;\Big|\; \Gamma = \int_0^{\infty} \text{Exp}\left\{ \left(g - k \right) t \right\} \delta t = \frac{1}{k-g} \text{ ...when... } m = \infty \tag{6}$$

Using Equations (5) and (6) above, the equation for enterprise value is...

$$V_0 = \Gamma\, C_0 \tag{7}$$

Annual Discounting

Question 1: What is the enterprise value of ABC Company given annual discounting?

Using Equation (2) above, the equation for the number of intraperiods within each annual discounting period is...

$$m = 1 \text{ ...such that... } \Delta = \frac{1}{m} = 1 \tag{8}$$

Using Equations (3), (4) and (8) above and the data in Table 1 above, the equations for the intraperiod cash flow growth rate and cost of capital are...

$$g = \left(1 + 0.0500 \right)^{1} - 1 = 0.0500 \text{ ...and... } k = \left(1 + 0.1500 \right)^{1} - 1 = 0.1500 \tag{9}$$

If annual cash flow grows that rate g and is discounted at rate k then using Equation (9) above, the value of the discount factor in Equation (5) above is...

$$\theta = \frac{1+g}{1+k} = \frac{1+0.0500}{1+0.1500} = 0.91304 \tag{10}$$

Using Equations (6) and (10) above, the value of the cash flow multiple is...

$$\Gamma = 1.0000 \times \frac{0.91304}{1 - 0.91304} = 10.50 \tag{11}$$

Using Equations (1), (7), and (11) above, the answer to the question is...

$$V_0 = \Gamma\, C_0 = 10.50 \times 1,000 = 10,500 \tag{12}$$

Monthly Discounting

Question 2: What is the enterprise value of ABC Company given monthly discounting?

Using Equation (2) above, the equation for the number of intraperiods within each annual discounting period is...

$$m = 12 \text{ ...such that... } \Delta = \frac{1}{m} = 0.0833 \tag{13}$$

Using Equations (3), (4) and (13) above and the data in Table 1 above, the equations for the intraperiod cash flow growth rate and cost of capital are...

$$g = \left(1 + 0.0500 \right)^{0.0833} - 1 = 0.00407 \text{ ...and... } k = \left(1 + 0.1500 \right)^{0.0833} - 1 = 0.01172 \tag{14}$$

If annual cash flow grows that rate g and is discounted at rate k then using Equation (14) above, the value of the discount factor in (5) above is...

$$\theta=\frac{1+g}{1+k}=\frac{1+0.00407}{1+0.01172}=0.99245 \tag{15}$$

Using Equations (6) and (15) above, the value of the cash flow multiple is...

$$\Gamma=0.0833\times\frac{0.99245}{1-0.99245}=10.95 \tag{16}$$

Using Equations (1), (7), and (16) above, the answer to the question is...

$$V_0=\Gamma\,C_0=10.95\times 1,000=10,950 \tag{17}$$

Continuous Discounting

Question 3: What is the enterprise value of ABC Company given continuous discounting?

Using Equation (2) above, the equation for the number of intraperiods within each annual discounting period is...

$$m=\infty\ \text{ ...such that... }\ \Delta=\frac{1}{m}=0 \tag{18}$$

Using Equations (3), (4) and (18) above and the data in Table 1 above, the equations for the intraperiod cash flow growth rate and cost of capital are...

$$g=\ln\Bigg(1+0.0500\Bigg)=0.04879\ \text{ ...and... }\ k=\ln\Bigg(1+0.1500\Bigg)=0.13976 \tag{19}$$

Using Equations (6) and (19) above, the value of the cash flow multiple is...

$$\Gamma=\frac{1}{0.13976-0.04879}=10.99 \tag{20}$$

Using Equations (1), (7), and (20) above, the answer to the question is...

$$V_0=\Gamma\,C_0=10.99\times 1,000=10,999 \tag{21}$$

References

[1] Gary Schurman, *Polylogarithm Of Order Zero,* May, 2019

[2] Gary Schurman, *The Dividend Discount Model,* May, 2019